Acceleration induced neutron emission in heavy nuclei

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Plan

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Motivation

What if, like a fully-filled water tank, a nucleus will spill its less bound nucleons when accelerated? Most probable this could happen to neutrons since, contrary to protons, they are not protected by a Coulomb barrier. In the eventuality that the answer is "yes", we are dealing with a new nuclear process and therefore its study should have high priority.

We choose a simple framework: the independent particle shell model without spin-orbit interaction. We solve the time-dependent Schrödinger equation with a moving mean-field of Woods-Saxon type. We study the evolution of few single-neutron states around the Fermi level during 10^{-21} sec of constant acceleration followed by 10^{-21} sec of constant velocity.

2D-TDSE with moving potential

We consider a neutron in a moving nuclear potential that has axial symmetry. It is represented by a wave function solution of the Schrödinger equation

$$i\hbar\frac{\partial\Theta(\rho,z,t)}{\partial t}=\mathcal{H}(\rho,z,t)\Theta(\rho,z,t), \qquad (1)$$

where $\mathcal{H}$ is the single-particle Hamiltonian.

$$\mathcal{H}=-\frac{\hbar^2}{2m}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{\partial^2}{\partial\rho^2}+\frac{\partial^2}{\partial z^2}-\frac{\Lambda^2}{\rho^2}\right]+V(\rho,z-\alpha(t)).$$

$\alpha(t)$ describes the displacement of the potential in time along the z axis. Λ is the projection of the orbital angular momentum on the symmetry axis. For simplicity the spin-orbit term is neglected.

By the Liouville transformation $\Phi = \rho^{1/2}\Theta$, the first derivative with respect to ρ from $\mathcal{H}$ is removed, resulting a simplified Hamiltonian H of the form:

$$H = -\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial\rho^2} + \frac{\partial^2}{\partial z^2} - \frac{\Lambda^2 - 1/4}{\rho^2}\right] + V(\rho, z - \alpha(t)).$$

One arrives to the equation

$$i\hbar\frac{\partial\Phi(\rho, z, t)}{\partial t} = H(\rho, z, t)\Phi(\rho, z, t). \qquad (2)$$

To solve this equation, a transformation of both the variable and the function from the non-inertial to the inertial system is very useful. It avoids an interpolation of the potential between the grid points at each time step.

Transformation from the fix to the moving frame

To introduce this transformation we will use the 1-D TDSE:

$$i\hbar \frac{\partial \Phi(t,z)}{\partial t} = -\frac{h^2}{2m}\frac{\partial^2 \Phi(t,z)}{\partial z^2} + V(z-\alpha(t))\Phi(t,z) \quad (3)$$

We go in the nuclear frame by the following changes of the variable $z \to q$ and of the function $\Phi \to \Psi$ (P. Rouchon, 2nd IFAC Workshop on Lagragian and Hamiltonian Methods for Nonlinear Control, Seville, 2003)

$$q = z - \alpha(t), \quad \Phi(t,z) = \exp(u)\Psi(t,q) \quad (4)$$

where

$$u = ib\left(z\dot{\alpha} - \alpha\dot{\alpha} + \frac{1}{2}\int_0^t \dot{\alpha}^2(t')dt'\right).$$

By taking $b = \frac{m}{\hbar}$, it can be shown that Eq.(3) will be transformed in

$$i\hbar \frac{\partial \Psi(t,q)}{\partial t} = -\frac{h^2}{2m}\frac{\partial^2 \Psi(t,q)}{\partial q^2} + V(q)\Psi(t,q) + mq\ddot{\alpha}(t)\Psi(t,q). \tag{5}$$

To eliminate the linear term in q (which tends to ∞ as $q \to \infty$), a further function transformation is performed

$$\Psi(t,q) = \exp\left(-i\frac{\lambda}{\hbar}\right)\chi(t,q) \tag{6}$$

with

$$\lambda(t,q) = qm\int_0^t \ddot{\alpha}(t')dt' = q\beta(t).$$

Particular case: $\alpha(t) = \frac{1}{2}At^2$

$$\dot{\alpha} = At, \ddot{\alpha} = A, \lambda = Bqt, B = qmA$$

$$u = ib\left(zAt - \frac{1}{3}A^2t^3\right)$$

$$i\hbar\frac{\partial\chi(t,q)}{\partial t} = -\frac{h^2}{2m}\left(\frac{\partial^2\chi(t,q)}{\partial q^2} + \frac{2}{\hbar i}Bt\frac{\partial\chi(t,q)}{\partial q} - \frac{1}{\hbar^2}B^2t^2\chi(t,q)\right) \tag{7}$$

$$+V(q)\chi(t,q).$$

The advantage of these transformations is that they lead to equations in which the potential depends on a time-independent variable, the dependence on $\alpha(t)$ being transferred in the coefficients of Eqs.(5),(7).

Numerical details

We solve numerically the equations (5) and (7). We work with the variables ρ and q on a finite numerical grid: [0,84]x[-256,256], $d\rho = dq =$1/8 fm, dt=1/128 $\times 10^{-22}$ sec. TDSE is solved by the Crank-Nicolson method. One obtains a linear system which is solved by a routine based on the Strong Implicit Procedure. C. R. Jesshope, *SIPSOL - suite of subprograms for the solution of the linear equations arising from elliptic partial differential equations*,Comp. Phys. Commun. **17** (1979) 383.

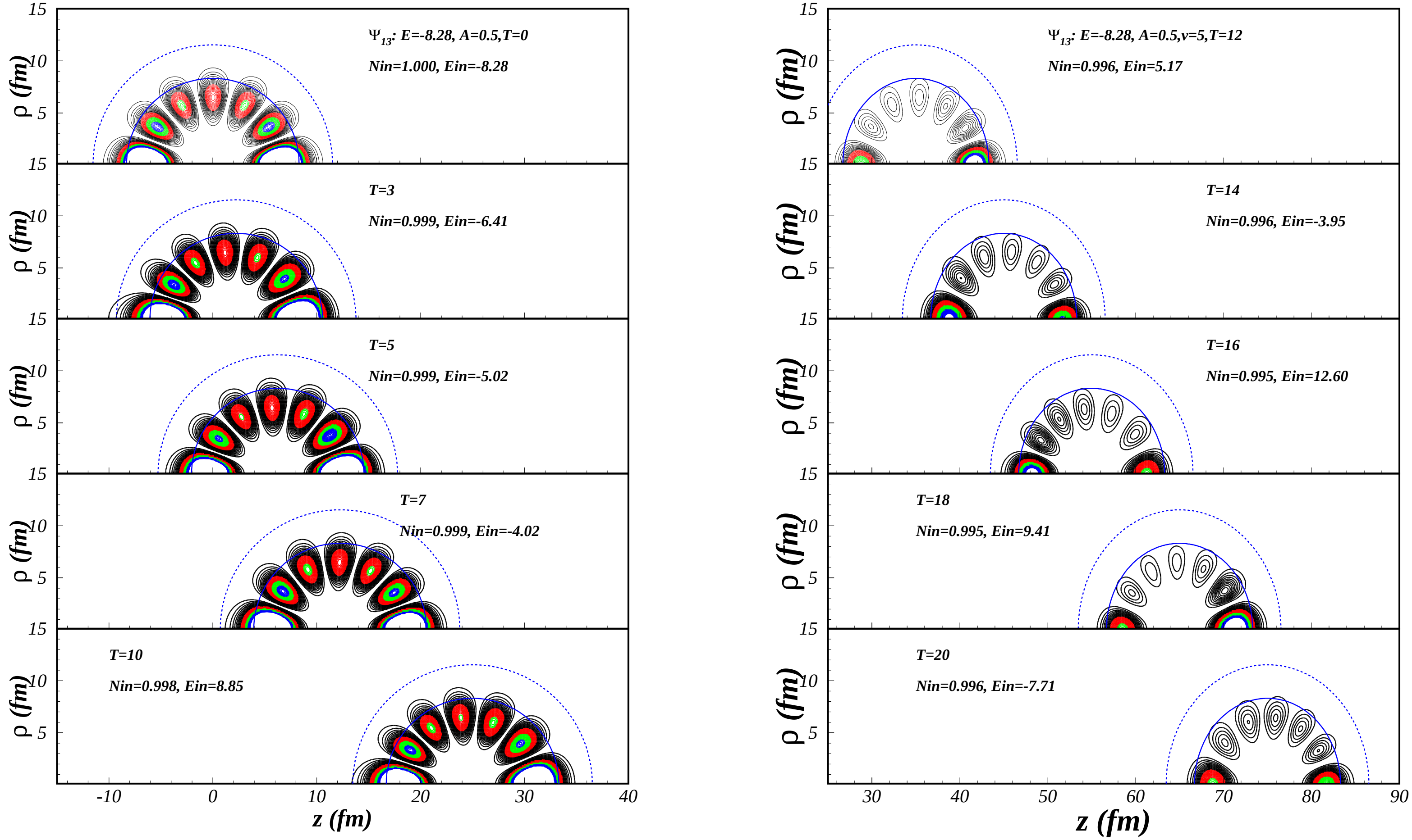
As initial solutions (at $t = 0$) we consider eigenfunctions of the original Hamiltonian.

As an example we take ^{236}U, a constant acceleration A=0.5 $[10^{44} fm/sec^2]$ and study 3 neutron states around the Fermi level. The constant velocity ν=5$[10^{22} fm/sec]$.

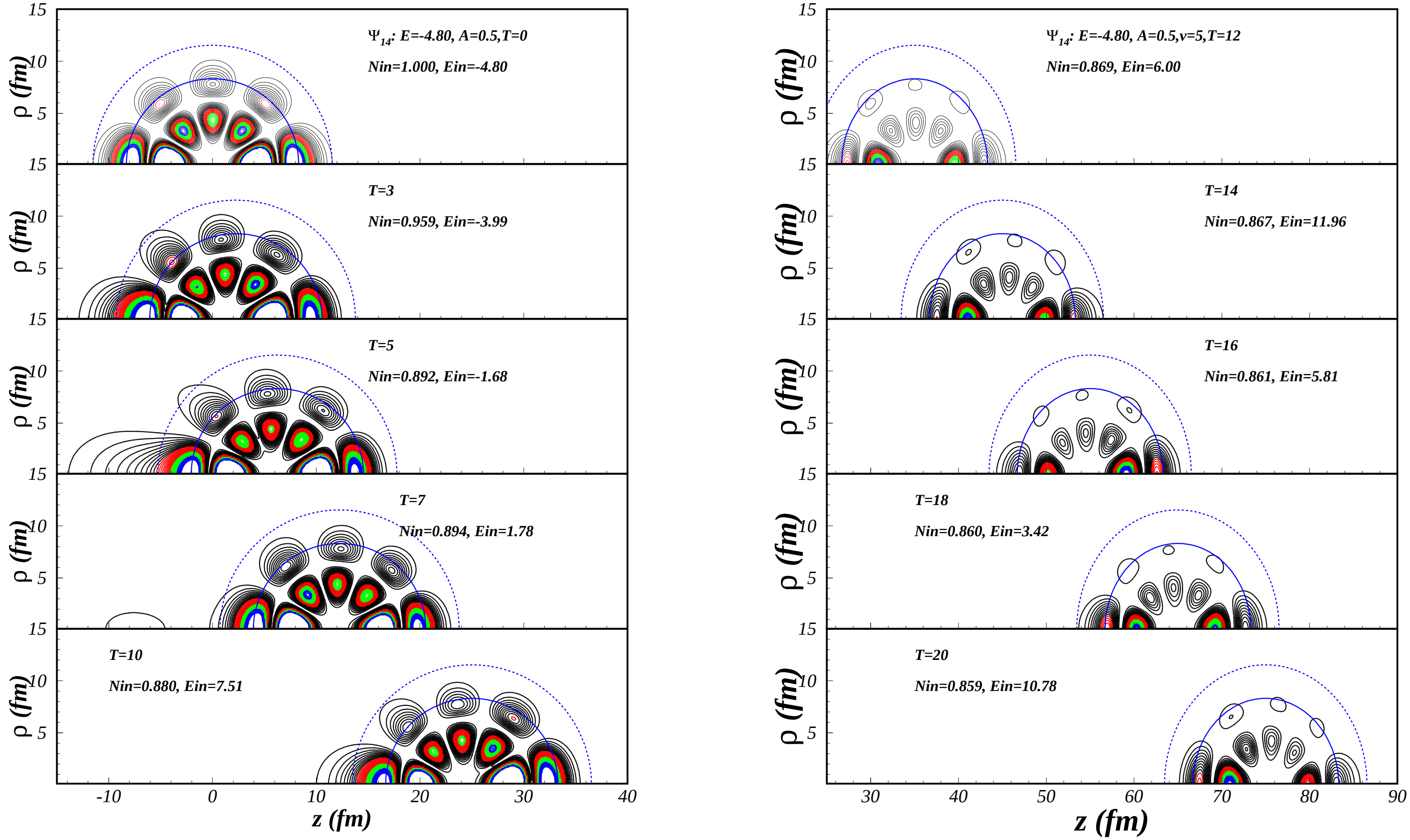
Time evolution during and after acceleration

Eigenfunction Φ^0_{13}; eigenvalue -8.28 MeV

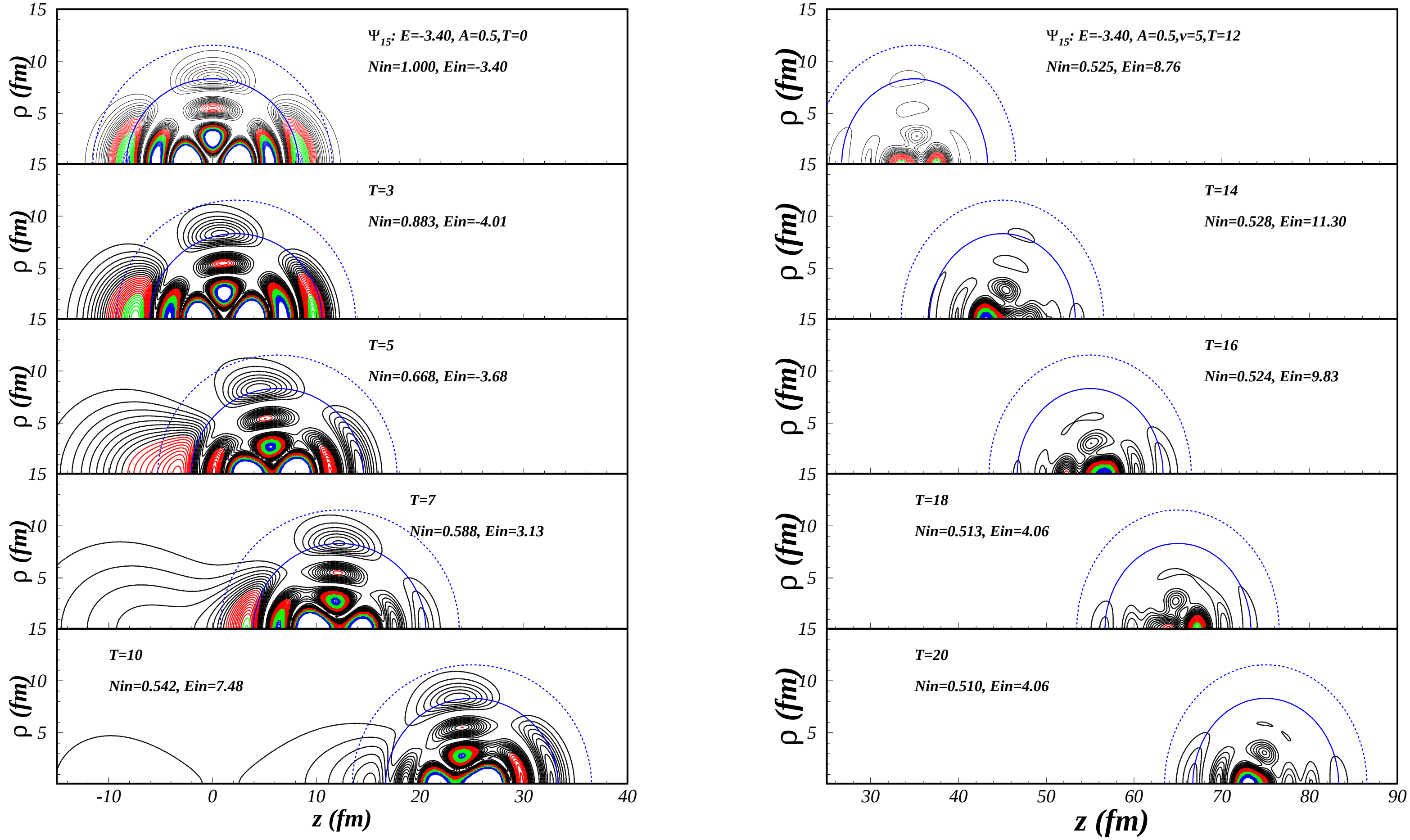
N_{in} is the norm inside the dotted sphere ($V_0/100$).



Eigenfunction Φ^{0}_{14}; eigenvalue -4.80 MeV .



Eigenfunction Φ^0_{15}; eigenvalue -3.40 MeV



Calculus of the average energy in the laboratory frame

$$\langle\Phi|H|\Phi\rangle = -\frac{h^2}{2m}\int\int\left(\Psi^*\frac{\partial^2\Psi}{\partial\rho^2}+\Psi^*\frac{\partial^2\Psi}{\partial q^2}\right)d\rho dq \qquad (8)$$

$$+\frac{h^2}{2m}b^2\dot{\alpha}^2\int\int|\Psi|^2 d\rho dq - \frac{h^2}{2m}2ib\dot{\alpha}\int\int\Psi^*\frac{\partial\Psi}{\partial q}d\rho dq$$

$$+\frac{h^2}{2m}\left(\Lambda^2-1/4\right)\int\int\frac{1}{\rho^2}|\Psi|^2 d\rho dq + \int\int V(\rho,q)|\Psi|^2 d\rho dq$$

The 1st term is the average kinetic energy in the nuclear frame. The 2nd term reduces to $m\dot{\alpha}^2/2$; it is the extra kinetic energy due to the velocity of the potential. The 3rd term reduces to $\dot{\alpha} < p >$ where $< p >$ is the average momentum in the nuclear frame. It represents the energy the neutron gains due to the interaction with the moving wall of the potential; equivalent to the "one-body" dissipation.

Discussions and Conclusions

A similar emission takes place during the slowing down (A < 0) of a projectile when it approaches a target. Among other effects, it will produce an increase of the neutron-transfer cross section. The value chosen (A=0.5) is larger than the acceleration during the Coulomb repulsion of two equal fission fragments from 236U separated by D_{cm}= 20 fm (A=0.13) but comparable with that attained during the collision of two 236U nuclei at the same distance of approach (A=0.52). However, the acceleration produced in Coulomb interactions is not constant (it depends on D_{cm}) and a high level cannot be maintained for 10^{-21} sec. A dedicated study is necessary. After more than a century since the nucleus was discovered, there are still new nuclear processes to be studied. This demonstrates how rich and diverse our field is.